Subspace Method for Blind Characterization of Atmospheric Scattering Model

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Abstract

This paper deals with the blind identification of a state-space model (A, B, C) characterizing the **Navier-Stokes** equation in an observed area and the localization of the source which emmits in this area. This identification is performed on the only basis of measurements provided by a distributed sensors network located in the monitored area. In order to solve the problem, subspace-based blind identification method joined to transport delays approximation is used to perform the evolution and control matrices (A and B) estimation problem. Furthermore, the state-space formulation brings *a priori* information which permits to reduce the number of undeterminated parameters in this subspace method. So, the aim of this contribution is to propose an identification and localization method based on a subspace approach combinated with *a priori* information about the form of the state-space matrices. Numerical simulations demonstrate the performance of the method in the case of a gas propagation.

Key words: Blind model identification, blind source localization, multisensor monitoring, environment.

1 Introduction

Thanks to the increasing number of supervision networks, the pollution is became better and better quantified, especially around the danger zones. Consequently the localization of source of pollution is a problem frequently posed. Let be a rectangular area monitored by a sensors network. Each of the sensor provides the concentration of a chemical component in the atmosphere. Let's suppose that the measurements show a huge variation which is due to an emission discontinuity from a known or unknown source. The problem is then to localize the source and to identify the propagation system between the source and the sensors only with the observed signals, and without any information on this source. So, blind identification methods are used, and especially the subspace one which permits to estimate the impulse responses of the source-sensors channels. The principle of this method is adapted to our problem thanks to *a priori* information brought by the state-space model characterizing the dispersion phenomenon. Indeed, the discretization of the **Navier-Stokes** equation leads to particular shapes of the matrices A and B. This information combinated with the subspace method reduces the number of undefined parameters. A term based on the intercorrelation of the observed signals is added to the criterion of the subspace method in order to perform the right localization.

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2 Scattering model : expression and discretization

The chosen model is **Navier-Stokes** differential equation, limited to the bidimensionnal case. This equation takes into account the two essential phenomena for the dispersion of a gas in the atmosphere. The first phenomenon is the advection depending on the wind, and the second is the diffusion depending on coefficients characterizing the turbulences, the sunniness and the temperature. The bidimensionnal approximation of the real 3D model can be realistic if the height difference between source and sensors is not significant with respect to the size of the observed area. The general equation is :

$$(1)\quad \frac{\partial \upsilon(x,y,t)}{\partial t} + U_x(t)\frac{\partial \upsilon(x,y,t)}{\partial x} + U_y(t)\frac{\partial \upsilon(x,y,t)}{\partial y} = \frac{\partial}{\partial x}(K_x(t)\frac{\partial \upsilon(x,y,t)}{\partial x}) + \frac{\partial}{\partial y}(K_y(t)\frac{\partial \upsilon(x,y,t)}{\partial y}) + S(x,y,t) + R(t)$$

where $\upsilon(x,y,t)$ is the gas concentration. To simplify the general model, some hypotheses are currently used :

- No chemical recombinations $\Rightarrow R(t) = 0$.
- The diffusion coefficients K_x and K_y are supposed temporally and spatially invariant.
- the wind vector $\overrightarrow{U}$ is completly defined and constant during the data acquisition.

In the real case, these hypotheses are a bit restrictive. Indeed, the model described has a good behaviour if atmospheric conditions (wind, temperature, ...) are constant or subject to low variations. So observation time can't exceed a few hours. The final equation is then :

$$(2)\qquad \frac{\partial \upsilon(x,y,t)}{\partial t} + U_x\frac{\partial \upsilon(x,y,t)}{\partial x} + U_y\frac{\partial \upsilon(x,y,t)}{\partial y} = K_x\frac{\partial^2 \upsilon(x,y,t)}{\partial x^2} + K_y\frac{\partial^2 \upsilon(x,y,t)}{\partial y^2} + S(x,y,t)$$

The numerical form of (2) is provided by the finite difference method called "upwind" [1] which takes into account the wind direction in the estimation of the first derivative :

$$(3)\qquad \Upsilon_{i,j}^{n+1} = m_1\Upsilon_{i+1,j}^n + m_2\Upsilon_{i,j}^n + m_3\Upsilon_{i-1,j}^n + m_4\Upsilon_{i,j+1}^n + m_5\Upsilon_{i,j-1}^n + m_6 S_{i,j}^n$$

with

$$(4)\qquad \begin{aligned} m_1 &= \frac{pK_x}{h^2} & m_2 &= 1 - \frac{p|U_x|}{h} - \frac{p|U_y|}{k} - \frac{2pK_x}{h^2} - \frac{2pK_y}{k^2} \\ m_3 &= \frac{p|U_x|}{h} + \frac{pK_x}{h^2} & m_4 &= \frac{pK_y}{k^2} \\ m_5 &= \frac{p|U_y|}{k} + \frac{pK_y}{k^2} & m_6 &= p \end{aligned}$$

$\Upsilon_{i,j}^n$ is now the discrete form at position (i,j) and time n of the continuous concentration $\upsilon(x,y,t)$. h, k are the spatial steps along x and y, and p is the sample period. The source being ponctual, the contribution of the source term is null everywhere except for the source position (x_s, y_s). After this step of discretization, the model is reformulated in a state-space form.

3 State-space representation

Without the source term, the equation (3) can be reformulated as $\Upsilon^{n+1} = A\Upsilon^n$. If we define a networking of size $((l/h) \times (L/k))$ (where $(l \times L)$ is the size of the area), then the state vector Υ^n is built by stacking the (L/k) rows of the networking one after the other. This implies that Υ^n has a $((l/h) \times (L/k)) \times 1$ size. This state vector coding leads to a block-tridiagonal form of the matrix A :

$$A = \begin{pmatrix} A_{1,1} & A_{1,2} & 0 & \cdots & \cdots & 0 \\ A_{2,1} & A_{2,2} & A_{2,3} & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & A_{(L/k)-1,(L/k)-2} & A_{(L/k)-1,(L/k)-1} & A_{(L/k)-1,(L/k)} \\ 0 & \cdots & \cdots & 0 & A_{(L/k)-1,(L/k)} & A_{(L/k),(L/k)} \end{pmatrix}$$

where each block has the form :

$$A_{k,k} = \begin{bmatrix} m_2 & m_1 & & \\ m_3 & m_2 & m_1 & \\ & \ddots & \ddots & \ddots \\ & m_3 & m_2 & m_1 \\ & & m_3 & m_2 \end{bmatrix} \qquad A_{k,k+1} = \begin{bmatrix} m_4 & & \\ & \ddots & \\ & & m_4 \end{bmatrix} \qquad A_{k,k-1} = \begin{bmatrix} m_5 & & \\ & \ddots & \\ & & m_5 \end{bmatrix}$$

The model stability is guaranteed if the eigenvalues of A are into the unit norm circle. This is possible if the relation (5) given by the **Gershgorin-Hadamard** theorem [2] is verified.

$$m_1 + m_2 + m_3 + m_4 + m_5 \leqslant 1 \tag{5}$$

By integrating the source term, we obtain a complete state-space model :

$$\begin{cases} \Upsilon^{n+1} & = A\Upsilon^n + m_6 B S^n + w^n \\ y^n & = C\Upsilon^n + b^n \end{cases} \tag{6}$$

The term $m_6 S^n$ represents the entry of the system, w^n models the background concentration, y^n is the measurement vector provided by the sensor network, and b^n is a white noise. So the estimation of this model leads to a characterization of the dispersion from a spatial and temporal point of view. Indeed, the matrix A is completly defined thanks to the m_q parameters which depend on the dispersion conditions, and the matrix B informs on the source position. The coding of the matrices B and C is the same than the state vector one. So, the matrix B is a full zero $((l/h) \times (L/k)) \times 1$ matrix except for the element corresponding to the source position which is set to 1 (*i.e* the $(((y_s/k-1)L/k)+x_s/h)th$ element). The matrix C uses the same principle for the sensors and is a $n_c \times ((l/h) \times (L/k))$ (n_c is the number of sensors).

$$\mathbf{B} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & 1 & 0 & \cdots & \cdots & \cdots & 0 \end{pmatrix}$$

So, the blind identification amounts to estimate the m_q $(1 \leqslant q \leqslant 5)$ parameters defining the matrix A for the propagation system and the matrix B for source position. This is possible thanks to the subspace method.

4 Blind identification method

The principle of the subspace method presented by Tong [5] and modified by Moulines [4] is to use the orthogonality property between the signal and noise subspaces in order to identify the impulse responses of channels linking the source to each sensor (Figure 1). This method only takes into account the sensor signals and their second order statistics. Let $\mathcal{H}_N$ be the $N \times (N+M)$ filtering matrix (M is the order of the filter) associated with the filter h_i^n :

$$\mathcal{H}_N = \begin{pmatrix} h_i^0 & \cdots & h_i^M & 0 & \cdots & 0 \\ 0 & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & h_i^0 & \cdots & h_i^M \end{pmatrix}$$

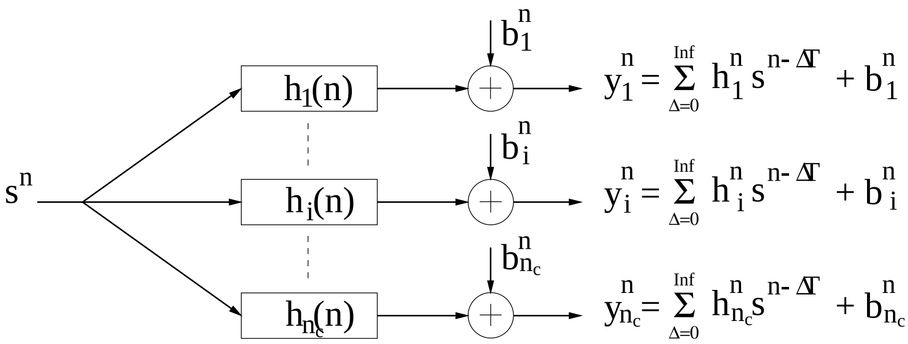


Figure 1: Channels between source and sensors

then, we can write the following expression in the multisensor case :

$$
\begin{array}{rcl}
\begin{bmatrix} Y_1^n \\ \vdots \\ Y_{n_c}^n \end{bmatrix} & = & \begin{bmatrix} \mathcal{H}_N^1 \\ \vdots \\ \mathcal{H}_N^{n_c} \end{bmatrix} S^n + \begin{bmatrix} \mathcal{B}_1^n \\ \vdots \\ \mathcal{B}_{n_c}^n \end{bmatrix} \\
Y^n & = & \mathcal{H}_N S^n + \mathcal{B}^n
\end{array} \tag{7}
$$

with $Y^n = [y^n, \cdots, y^{n-N+1}]^T$, $\mathcal{B}^n = [b^n, \cdots, b^{n-N+1}]^T$. Under a few identifiability conditions [4] and if the source is statistically independant of the noise, then the autocorrelation matrix of the observed signals is :

$$R_y = E[Y^n Y^{nT}] = \mathcal{H}_N R_s \mathcal{H}_N^T + R_b \tag{8}$$

If we assume that $R_b = \sigma^2 I$ (white noise with variance σ^2), then the EVD of R_y gives, after a classification of the eigenvalues and of their associated eigenvetors in a decreasing order, the subspace decomposition :

$$R_y = E[Y^n Y^{nT}] = U diag(\lambda_0, \lambda_1, \cdots, \lambda_{M+N-1}) U^H + \sigma^2 V I V^H \tag{9}$$

This decomposition gives the signal subspace U and the noise subspace V (the order M of the largest filter is given by the size of U). The signal subspace is the same than the one generated by the columns of $\mathcal{H}_N$, so the orthogonality property is also available between the columns of $\mathcal{H}_N$ and the noise subspace. On this basis, a criterion has been defined to identify the channels parameters :

$$J(H) = H^T Q H \tag{10}$$

with $Q = \sum_{i=0}^{n_c N - M - N - 1} \mathcal{V}_i \mathcal{V}_i^T$ ($\mathcal{V}_i$ is the filtering matrix associated with the vector V_i), and $H = [H_1^T, \cdots, H_{n_c}^T]^T$. The minimisation is computed under the constraint $|H| = 1$.

5 Localization

If we consider the problem as just before, we must estimate $n_c M$ parameters to identify the impulse response. By integrating the *a priori* information brought by the state-space matrices, we can reduce the number of parameters to two (for a given source position). Indeed, in the equation (2), the only undefined parameters are K_x and K_y. We shown that $A = f(m_q)$ (6) and $m_q = g(K_x, K_y)$ (4), so A depends on K_x and K_y. By the relationship (11) between the impulse response and the state-space matrices

$$H^i = [C^{(i)} B, \cdots, C^{(i)} A^k B, \cdots, C^{(i)} A^{M-1} B]^T \tag{11}$$

(with $C^{(i)}$ the row i of the matrix C), it's easy to show that H depends on B, K_x, K_y. For a given source position, $H = \mathcal{F}(K_x, K_y)$. It's important to note that the minimisation of $J(H(B, K_x, K_y))$ is no more performed with the constraint $|H| = 1$, but with the parametrized form of H (11). Nevertheless, H is estimated up to transport delays for each impulse response. So, propagation delays estimation is needed to perform the source localization.

Let τ_{H^i} be the transport delay of the impulse response H^i (*i.e* $H^i(0) = \cdots = H^i(\tau_{H^i}) = 0$). By the second order statistics, transport delay can't be estimated. At most, it is only possible to approximate the time delay $(\tau_{H^i} - \tau_{H^j})$. When sensors layout permits to ensure the channels disparity (no common zeros after the transport delay), the time delay ϕ_{ij} between two sensors measurements depend on the time delay $\tau_{H^i} - \tau_{H^j}$ and on the filters phase difference. Assuming that the source signal frequency is in the passband of each channel, we can consider that the filters phase difference is negligible. Then, the observable time delay between two sensor signals corresponds to the one induced by the difference $\tau_{H^i} - \tau_{H^j}$. So, the following expression can be written :

$$\widehat{\tau_{H^i} - \tau_{H^j}} \approx arg\,\max_{\tau}(r_{y_i y_j}(\tau)) \tag{12}$$

where $r_{y_i y_j}(\tau)$ is the crosscorrelation sequence. This information is added to the subspace method in order to carry out the complete characterization.

So, finding the best localization of the source B^* leads to a minimisation of the criterion (10) under the constraint :

$$J'(B) = \sum_{i,j}^{n_c}(\tau_{H^i}(B) - \tau_{H^j}(B)) - arg\,\max_{\tau}(r_{y_i y_j}(\tau)) \tag{13}$$

Let's note that the constraint $J'(B)$ is null for $B = B^*$ in the case where source signal is sinusoidal, but generally $J'(B)$ is just an approximation of the matching time delay. Finally, solving the both problem of localization and identification consist in minimizing the criterion :

$$B^* = arg\,\min_{B}(H(K_x(B), K_y(B), B)^T Q H(K_x(B), K_y(B), B) + J'(B)) \tag{14}$$

The right loacalization B^* gives the best coefficients $K_x(B^*)$ and $K_y(B^*)$. The minimisation of this criterion is made by the mean of a levenberg-Marquart procedure [3]. We can see that this method needs a great computation time because (14) is calculated for all the possible source positions, *i.e* all the point of the networking except the sensors positions.

6 Results

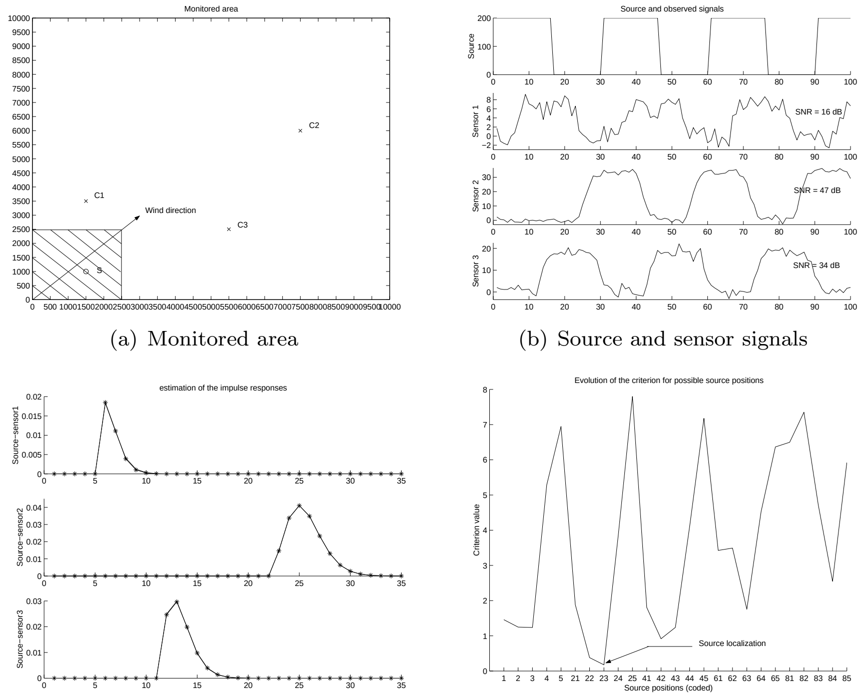
The simulation conditions are :

- $\overrightarrow{U}$: $(6m/s, 45°)$.
- size of monitored area $(10km, 10km)$, $h = 500m$, $k = 500m$
- source coordinates : $(1500m, 1000m)$
- sensors coordinates : C1 : $(1500m, 3500m)$, C2 : $(7500m, 6000m)$, C3 : $(5500m, 2500m)$
- noise : $\sigma_b^2 = 2.25$

A subarea has been defined thanks to the knowledge of the wind direction and to the hypothesis that the source is located before the sensors. This subarea permits to reduce the computation time (Figure 2.a).

7 Conclusion

In this contribution we have proposed a method of characterization of a scattering model acting in a monitored area and including a source. The subspace method permits the identification without any *a priori* information (spatial and temporal) on the source. The estimation of the impulse responses of each source-sensors channel is based on the minimisation of a criterion which takes into account the orthogonality property of the two subspaces and the estimation of the delays between the sensor signals. The algorithm provides good results between 20dB and 50 dB. Indeed, above 50dB the noise subspace does not bring sufficient information and under 20 dB, the delays estimation is not good.



(a) Monitored area

(b) Source and sensor signals

(c) Impulse responses : originals (–) and estimated (-*-)

(d) Criterion (14) evolution

Figure 2: Simulation results

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